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JPRS: 5519

9 September 1960

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- USSR

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~~Price: $0.50~~

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JPRS: 5519
CSO: 4361-N/b

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/Following is the translation of an article by E. I. Andriankin entitled "Skhodyashchayasya Volna v Plasticheskoy Srede" (English version above) in Doklady Akademii Nauk SSSR (Proc. Acad. Sci. USSR) Vol. ~~59~~, No. 4, ~~1948~~, pages ~~659-662~~/ 131
1960 769-772

(Presented by Academician N. N. Semenov, 13 November 1959)

The problem of an explosion in a plastic compressible medium has been investigated by A. S. Kompaneyets in /1/. In this work a simplified law of compression was assumed. This made it possible to obtain simple equations for the law of motion of the front of a diverging spherical wave.

Utilizing the analogous law of compression (we will assume that the density of the medium reaches the limit value $\rho_* > \rho_0$ at any pressure different from zero), we will consider the problem of the converging plastic wave, which can be formulated as follows.

At the instant $t = \tau_0$, let there be a pressure at the free surface of a spherical layer with an initial outside radius a_0. Let the said pressure vary according to a given pattern $P = P_0 F(t/\tau_0)$ (in a more general case $F = F(t/\tau_0, x)$ can be assumed). At the inner surface of the spherical layer of initial radius b_0 the pressure is zero. At the instant the pressure is applied, a shock wave is propagated through the material.

We assume the medium behind the front to be incom-

pressible, $\rho_f = \rho_* > \rho_0$, and the condition of plasticity to be satisfied:

$$\sigma_r - \sigma_\theta = k + m(\sigma_r + 2\sigma_\theta). \tag{1}$$

where k and m are assumed to be known constants, σ_r and $\sigma_\theta = \sigma_\varphi$ are the main stresses. The subscript f denotes the values at the wavefront. Further, we assume that the initial density of the material in the spherical layer depends on the radius $\rho_0 = \rho_0(r_0)$.

We shall solve the problem in Lagrangian variables, where the equations of continuity will be:

$$\frac{\partial r}{\partial r_0} = \frac{r_0^2 \rho_0(r_0)}{r^2 \rho}, \tag{2}$$

$$\rho \frac{\partial u}{\partial t} = \frac{\partial r}{\partial r_0} \frac{\partial \sigma_r}{\partial r_0} - \frac{2(\sigma_r - \sigma_\theta)}{r}. \tag{3}$$

Here $u = \partial r/\partial t = \dot{r}$, t is time, r and r_0 are the variable and initial coordinates of the particles.

Using (1) and (2), we can bring Eq. (3) into the form:

$$\frac{\partial}{\partial r_0}\left[r^\alpha\left(\frac{k}{3m} - P\right)\right] = \rho_0(r_0)\, r_0^2 r^{\alpha-2} \frac{\partial u}{\partial t}, \tag{4}$$

$$\alpha = -\frac{6m}{2m+1} \leqslant 0, \quad P = -\sigma_r.$$

The boundary conditions for Eqs. (2) and (4) are the laws of conservation at the wavefront, the equality of pressure at the free surface $r_0 = a_0$, and the condition of continuity of the variable radius

$$P_\Phi = \rho_0(R)\,\varepsilon_\Phi \dot{R}^2, \quad u_\Phi = \varepsilon_\Phi \dot{R}, \quad r_\Phi = R = r_0,$$
$$P(a_0, t) = P_0 F(t/\tau_0), \quad \varepsilon_\Phi = 1 - \rho_0/\rho_*. \tag{5}$$

We shall introduce the dimensionless parameters

$$a = a_*/a_0, \quad b = b_*/a_0, \quad x = R/a_0, \quad s = r_0/a_0, \quad \bar{r} = r/a_0,$$
$$\bar{\rho} = \rho/\rho_*, \quad \varkappa = k/3mP_0, \quad \bar{k} = k/P_0, \quad \tau = t/\tau_0, \quad \bar{u} = \partial\bar{r}/\partial\tau, \tag{6}$$
$$dx/d\tau = -\sqrt{y}, \quad y = \rho_* \dot{R}^2/P_0, \quad \tau_0 = a_0\sqrt{\rho_*/P_0}.$$

where a_* and b_* are the outside and inside radii of the spherical layer at the instant t.

Integrating (2) and using Conditions (5), we shall find

$$\bar{r}^3 = x^3 + 3\int_x^s \bar{\rho}_0(s)\, s^2 ds, \quad a^3 = x^3 + 3\int_x^1 \bar{\rho}_0 s^2 ds,$$
$$\bar{u} = -\frac{\lambda(x)}{\bar{r}^2}, \quad \lambda = \varepsilon_\phi x^2 \sqrt{y(x)}, \quad \varepsilon_\phi = 1 - \bar{\rho}_0(x). \tag{7}$$

Integrating Eq. (4) and using (7), we get

$$\bar{r}^\alpha \bar{p} = \varkappa(\bar{r}^\alpha - x^\alpha) + x^\alpha \bar{P}_\phi - \frac{d\lambda}{dx}\sqrt{y}\int_x^s \bar{\rho}_0(s)\, \bar{r}^{\alpha-4} s^2 ds + 2\lambda^2 \int_x^s \bar{\rho}_0\, \bar{r}^{\alpha-7} s^2 ds,$$

$$\bar{P}_\phi = \bar{\rho}_0(x)\, \varepsilon_\phi y, \quad x \neq 0. \tag{8}$$

If the law of motion of the wavefront is known, the pressure and velocity distribution in the entire region $x \leqslant s \leqslant 1$.

An ordinary differential equation for the velocity of the wavefront can be obtained from (8), if we use the condition on the free surface. Assuming also $\rho_0 = \rho_1 s^n$, while calculating all integrals in Eqs. (7) and (8), we shall find:

$$\frac{dz}{dx} = K(x)z + N(x,\tau), \qquad \frac{d\tau}{dx} = -\frac{x^2 \varepsilon(x)}{\sqrt{\varepsilon_1 z}}, \quad \tau = 1, \quad x = z = 1. \tag{9}$$

Here

$$z = \frac{y x^4 \varepsilon^2}{\varepsilon_1}, \quad \varepsilon = 1 - \beta x^n, \quad \varepsilon_1 = 1 - \beta, \quad \rho_0(a_0) = \beta\rho_*,$$

$$K = \frac{4(\alpha-1)x^2\varepsilon\,(a^{\alpha-4} - x^{\alpha-4})}{(\alpha-4)(a^{\alpha-1} - x^{\alpha-1})} + \frac{2\beta(\alpha-1)\,x^{n+\alpha-2}}{a^{\alpha-1} - x^{\alpha-1}},$$

$$a = \left[x^3 + \frac{3\beta(1 - x^{n+3})}{n+3}\right]^{1/3}.$$

$$N = \frac{2\beta(\alpha-1)x^2\varepsilon\,[\delta_1 - a^\alpha F(\tau, x)]}{\varepsilon_*(a^{\alpha-1} - x^{\alpha-1})}, \quad \delta_1 = \varkappa(a^\alpha - x^\alpha).$$

In order to integrate Eq. (9) it is necessary to know its asymptotic expression at $x \to 0$ and the value of the derivative dz/dx at point $x = 1$.

After a series of calculations, we shall find

$$3\left(\frac{dz}{dx}\right)_1 = 4\varepsilon_1 + 8 - 2\sqrt{\varepsilon_1}\left(\frac{dF}{d\tau}\right)_1 + \frac{2}{\varepsilon_1\beta}\left(\frac{d\varepsilon}{dx}\right)_1 + \delta_2, \quad \delta_2 = 2\alpha\beta(\alpha - 1); \tag{10}$$

$$\lim_{x\to 0}\tau = A, \quad z = B_1 x^{4(\alpha-1)/(\alpha-1)} + \ldots \quad (x \to 0,\ n > 0); \tag{11}$$

$$z = B_2 x^{\omega} + \ldots, \quad \omega = \frac{2(\alpha - 1)[2\varepsilon_1 - \beta(\alpha - 4)]}{\alpha - 4} \quad (n = 0,\ \alpha \leqslant 0). \tag{12}$$

However, if the outside pressure drops rapidly, the wave may stop before reaching the center.

Equations (8) and (9) are true in case $\alpha \neq 0$; if m = 0, the condition of plasticity becomes $\sigma_r - \sigma_\theta = k$ and the solution is given by the same equations, in which we have to assume

$$\alpha = 0, \quad \delta_1 = 2\bar{k}\ln(x/a), \quad \delta_2 = -4\beta\bar{k}.$$

It should also be noted that Eqs. (9) give a solution to the problem of the collapse of a spherical layer of incompressible plastic material. In that case it is necessary to assume

$$r^3 = s^3 + x^3 - b_0^3, \quad x = b, \quad \bar{P}_\phi = 0, \quad z = yx^4,$$
$$k = \frac{4(\alpha - 1)x^2(a^{\alpha-4} - x^{\alpha-4})}{(\alpha - 4)(a^{\alpha-1} - x^{\alpha-1})}, \quad N = \frac{2(\alpha - 1)x^2[\delta_1 - a^{\alpha}F]}{a^{\alpha-1} - x^{\alpha-1}}. \tag{13}$$

In these cases an asymptotic solution is given by Eq. (11). At $\tau = 1$ and $x = b_0$, the boundary conditions for this problem will be y = 0; in the case of initial motion in the fluid, the boundary condition will be $y = y_0$. The latter occurs, for instance, when a spherical layer of compressible material has been compressed by the outside pressure to the limit condition $\rho_f = \rho_*$; then, after the disappearance of the shock wave (at the boundary with the empty space) the compressed material continues to collapse as if incompressible.

In Figs. 1 and 2 the results of the numerical integration of Eqs. (8) are shown. Curve 1 corresponds to

the solution of the problem, for parameter values of:

$F = t^{-1.2}$

$\varepsilon_* = 0.1$

$n = 0$

$\chi = -0.1$

$\alpha = -1$

for Curve 2, we have:

$F = t^{-3}$

$\varepsilon_* = 0.1$

$n = 0$

$\chi = -0.1$

$\alpha = 0$

for Curve 3, we have:

$F = 1$

$\varepsilon_* = 0.2$

$n = 1$

$\beta = 0.8$

$\chi = -0.01$

$\alpha = -1.$

(In the case of a constant outside pressure, or $F = F(x)$, Eqs. (8) will be integrated in squares).

The values of A and B in the Asymptotic Expressions (11) and (12) can be determined by numerical calculation. In case 2, $A \approx 0.44$, $B \approx 1.195$, $\omega \approx 1.9$; in case 3, $A \approx 0.166$, $B \approx 1.464$, and $\omega \approx 1.6$.

The concentration of the energy in the center can be investigated through the behavior of the solution at $x \to 0$. The energy per unit volume consists of kinetic energy, dissipation energy at the wavefront, and dissipation energy due to the work of the forces of plastic deformation; therefore

$$\int_a^1 a^2 F\,da = \frac{1}{2}\int_x^1 \bar\rho_0(s)\,\bar u^2 s^2\,ds + \frac{1}{2}\int_x^1 \varepsilon_\Phi P_\Phi s^2\,ds + 2\int_1^{\bar\tau}\left[\int_x^1 \frac{(\bar\sigma_r - \bar\sigma_\theta)(1-\varepsilon_\Phi)\bar u s^2\,ds}{\bar r}\right]d\tau. \qquad (14)$$

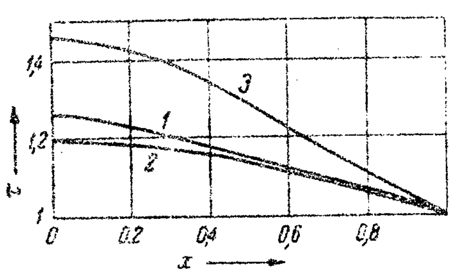

Fig. 1

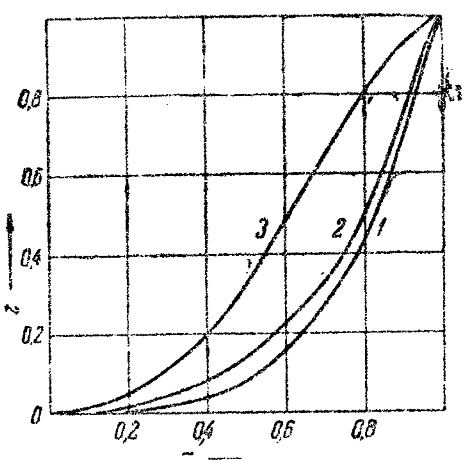

Fig. 2

It can be established, studying the behavior of these integrals at $x \to 0$ and $s \to 0$, that the final concentration of energy in the center takes place only in the case of the collapse of a spherical layer of incompressible fluid (the second integral in (14) is absent) under the condition $n = mk = 0$ ($\bar k$ is arbitrary). In other cases all integrals in (14) converge, and this means a decrease in the stored energy at the center.

In analogy to the above, the problem can be solved by assuming a variable compression of matter, depending on the amplitude of the pressure at the wavefront /2/. However, as in /3, 4/ it would be necessary here to solve an integral differential equation for the velocity of the front of the shock wave.

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Submitted 13 Nov. 19..

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